Name: ______________________________

CSE 5500 Algorithms

Exam II-B; November 27, 2018

Note: You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (17 points) Present an $O(n \log n)$ time algorithm to compute $f(x) = \Pi_{i=1}^{\log n}(x + a_i)^{2^i}$, where $a_1, a_2, \ldots, a_{\log n}$ are scalars. The coefficients of $f(x)$ should be output.

2. (17 points) X and Y are two binary strings with n and m bits, respectively (with $n > m$). The problem is to find all the occurrences of Y in X. Present an $O(n \log n)$ time algorithm for this problem. (You cannot state or use any known results on string matching for this problem.)

3. (16 points) Present an algorithm for finding a minimum cost spanning tree (MCST) of a given connected undirected weighted graph $G(V, E)$. The weight on each edge is w. Your algorithm should run in time $O(|E|)$. What is the total weight of the MCST?

4. (17 points) Input is a directed graph $G(V, E)$ where each edge has the same weight. The problem is to solve the all source shortest paths problem. Show how this can be done in $O(|V|^2 + |V|\ |E|)$ time.

5. (17 points) Let $A_n = \{a_1, a_2, \ldots, a_n\}$ be a finite set of distinct coin types (for example, $a_1 = 50¢$, $a_2 = 25¢$, $a_3 = 10¢$, and so on.) We can assume each a_i is an integer and $a_1 > a_2 > \cdots > a_n$. Each type is available in unlimited quantity. The coin-changing problem is to take an integer C as input and make up an exact amount C using a minimum total number of coins. Assume that $a_n = 1$ so that there is always a solution. Present an $O(Cn)$ time algorithm for this problem. *Hint: Use dynamic programming.*

6. (16 points) Let $G(V, E)$ a flow network with $V = \{s, a, b, c, d, t\}$, where s is the source and t is the sink. Edge capacities are: $c(s, a) = 10, c(s, c) = 10, c(a, b) = 8, c(a, c) = 6, c(b, a) = 5, c(b, d) = 9, c(b, t) = 8, c(c, a) = 3, c(c, d) = 4, c(d, b) = 5, c(d, c) = 6$, and $c(d, t) = 7$. Find the maxflow for G.