Name: ________________________________________

# CSE 5500 Algorithms

## Model Exam II, Fall 2018

**Note:** You are supposed to give proofs to the time bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (16 points) Present an $O(n \log^2 n)$ time algorithm to compute all the roots of a given degree-$n$ polynomial $f(x)$. Assume the following: 1) The roots of $f(x)$ are integers in the range $[1, cn]$ where $c$ is a constant; 2) The polynomial is given in coefficients form. (Recall that $a$ is a root of $f(x)$ if $f(a) = 0$.)

2. (18 points) Input is a sequence $X$ of pairs of real numbers $(r_1, a_1), (r_2, a_2), \ldots, (r_n, a_n)$. The problem is to find a polynomial $f(x)$ of minimum degree ($d$) such that $f(r_i) = a_i$, for $1 \leq i \leq n$. For example, if the input sequence is $(0, 1), (1, 3), (2, 5), (3, 7), (4, 9)$, the answer is $f(x) = 2x + 1$. Present an algorithm to solve this problem that runs in $O(n \log^3 d)$ time. (Assume that $n > d \log d$).

3. (17 points) Fibonacci heap is a data structure that can perform the following operations in $O(\log n)$ time each: FindMin, Insert, and DeleteMin. Here $n$ is the number of elements in the heap. It can also perform the DecreaseKey$(k, k')$ operation in $O(1)$ time. (This operation decreases the value of key $k$ by $k'$). Show that if we use this data structure in Prim's algorithm, its run time will be $O(|V| \log |V| + E)$.

4. (16 points) Input is an undirected graph $G(V, E)$. Present an $O(|V|^2)$ time algorithm to compute the reflexive transitive closure matrix $A^*$ of $G$. (Recall that $A^*[i, j] = 1$ iff there is a path from node $i$ to node $j$ in $G$.)

5. (17 points) Given are three strings $x, y$, and $z$ where $x = x_1 x_2 \ldots x_m$, $y = y_1 y_2 \ldots y_n$, and $z = z_1 z_2 \ldots z_{m+n}$. Give an $O(mn)$ time dynamic programming algorithm to check if $z$ is a *shuffle* of $x$ and $y$. (For example, if $x = x_1 x_2 x_3$, and $y = y_1 y_2$, then, $x_1 x_2 y_1 y_2 x_3$, and $y_1 x_1 y_2 x_2 x_3$ are (two of the) shuffles of $x$ and $y$, but $x_2 x_1 y_1 y_2 x_3$ is not a shuffle.)

6. (16 points) Consider a flow network $G(V, E)$ with $V = \{s, a, b, c, d, t\}$ where $s$ and $t$ are the source and the sink, respectively. $c(s, a) = 8, c(s, c) = 10, c(a, b) = 8, c(b, a) = 3, c(a, c) = 7, c(c, a) = 5, c(c, d) = 7, c(d, c) = 2, c(b, c) = 6, c(d, b) = 3, c(b, t) = 4$, and $c(d, t) = 10$. Use the Ford-Fulkerson algorithm to find a maximum flow for $G$. Show the steps of your algorithm.