Name: ____

CSE 5500 Algorithms

Exam III - Model; December, 2018

Note: You are supposed to give proofs to the time and processor bounds of your algorithms. Read the questions carefully before attempting to solve them.

1. (20 points) Input is an undirected graph $G(V, E)$ in the form of an adjacency matrix. The problem is to check if G is complete or not. Present an $O(1)$ time algorithm for this problem that uses at most $|V|^2$ common CRCW PRAM processors.

2. (20 points) Input are a sequence $X = r_1, r_2, \ldots, r_n$ of real numbers and an integer k. The problem is to output $s_1, s_2, \ldots, s_n$ where $s_i = \sum_{q=0}^{k-1} r_{i+q}$, for $i = 1, 2, \ldots, (n-k+1)$ and $s_i = r_i$ for $i = (n-k+2), (n-k+3), \ldots, n$. Present an $O(\log n)$-time algorithm for this problem that uses at most $\frac{n}{\log n}$ CREW PRAM processors.

3. (20 points) $\mathcal{A}$ is a parallel selection algorithm that runs in $O(\log n)$ time using $\frac{n}{\log n}$ CREW PRAM processors, on any n given arbitrary real numbers. Show how to employ $\mathcal{A}$ to sort n given arbitrary real numbers in $O(\log^2 n)$ time using $\frac{n}{\log n}$ CREW PRAM processors.

4. (20 points) The decision version of the zero-one knapsack problem takes as input n objects where each object has a profit and a weight. Input are also m and P where m is the capacity of the knapsack and P is a target profit. The answer is "yes" if there exists a subset of the objects whose total weight is $\leq m$ and whose total profit is $\geq P$; the answer is "no" otherwise. Assume that ZeroOneK is a polynomial time algorithm to solve the decision version of the zero-one knapsack problem. Show how you will use ZeroOneK to find a subset of the objects whose total profit is $\geq P$ and whose total weight is $\leq m$ if there exists one such subset. Your algorithm should run in polynomial time.

5. (20 points) Consider the following problem:

   > Input is a Boolean formula F in conjunctive normal form on n variables. The problem is to check if F has a satisfying assignment in which all but c variables have the value T. Here c is some constant.

   Is the above problem in $\mathcal{P}$? If so prove it, else prove that it is $\mathcal{NP}$-complete.