CSE 3500 Algorithms and Complexity

Homework 3: Due on December 3, 2026, 2 PM

1. Present a common CRCW PRAM algorithm that finds the maximum of n arbitrary elements in $O(1)$ time using $n^{1+\epsilon}$ processors for any fixed $\epsilon > 0$.

2. Present a CREW PRAM algorithm to multiply two given $n \times n$ matrices in $O(\log n)$ time. What is the processor bound?

3. Let $G = (V, E)$ be a directed graph with n vertices, represented by its $n \times n$ adjacency matrix A. Assume that n is a power of 2. Present a CREW PRAM algorithm to compute the transitive closure of G. In other words, compute an $n \times n$ Boolean matrix T such that $T[i, j] = 1$ if there is a directed path from i to j, and 0 otherwise. Your algorithm should run in $O(\log^2 n)$ time using $O(n^3)$ CREW PRAM processors. Analyze its time and processor bounds.

4. Let π_2 be a problem for which there exists a deterministic algorithm that runs in time $2^{\sqrt{n}}$ (where n is the input size). Prove or disprove:

   > If π_1 is another problem such that π_1 is polynomially reducible to π_2, then π_1 can be solved in deterministic $O(2^{\sqrt{n}})$ time on any input of size n.

5. Assume that there is a polynomial time algorithm CLQ to solve the CLIQUE decision problem. Show how to use CLQ to determine the maximum clique size of a given graph in polynomial time.

6. Consider the following problem, called Set Splitting:

   Input: A finite set S and a collection of subsets $C_1, C_2, \ldots, C_m$ of S, where each subset contains either two or three elements.

   Question: Can S be partitioned into two disjoint sets S_1 and S_2 such that every C_i contains at least one element from S_1 and at least one element from S_2?

   Prove that Set Splitting is NP-complete by giving a polynomial-time reduction from 3-SAT.